Chapter 9: Integration

Exercise 9g

(1) Let $I = \int 4x^3 e^{x^4}\, dx$

put $u = x^4$, $\therefore \frac{du}{dx} = 4x^3$ and $\therefore du = 4x^3\, dx$

So $I = \int e^u\, du = e^u + c$

$= e^{x^4} + c$

(2) let $I = \int \sin x \, e^{\cos x}\, dx$

put $u = \cos x$, $\therefore \frac{du}{dx} = -\sin x$ and $\therefore -du = \sin x\, dx$

Hence $I = \int -e^u\, du = -e^{\cos x} + c$

(3) Let $I = \int \sec^2 x . e^{\tan x}\, dx$

put $u = \tan x$, $\therefore \frac{du}{dx} = \sec^2 x$ and $\therefore du = \sec^2 x\, dx$

Hence $I = \int e^u \, du = e^u + c$

$= e^{\tan x} + c$

(4) Let $I = \int (2x+1) e^{(x^2+x)} \, dx$

Put $u = x^2 + x$, $\therefore \frac{du}{dx} = 2x+1$, and $\therefore du = (2x+1)dx$

Hence $I = \int e^u \, du = e^u + c$

$= e^{x^2+x} + c$

(5) Let $I = \int \operatorname{cosec}^2 x \cdot e^{(1-\cot x)} \, dx$

Put $u = 1 - \cot x$, so $\frac{du}{dx} = \operatorname{cosec}^2 x$

$\Rightarrow du = \operatorname{cosec}^2 x \, dx$

$\therefore I = \int e^u \, du = e^u + c$

$= e^{(1-\cot x)} + c$

(6) Let $I = \int x(x^2-3) \, dx$

Put $u = x^2 - 3$, $\therefore \frac{du}{dx} = 2x \Rightarrow \frac{1}{2} du = x \, dx$

So $I = \int \frac{1}{2} u \, du = \frac{1}{2} \frac{u^2}{2} + c = \frac{1}{4}(x^2-3)^2 + c$

(7) Let $I = \int x\sqrt{1-x^2}\,dx$

put $u = 1-x^2$, $\therefore \frac{du}{dx} = -2x \Rightarrow -\frac{1}{2}du = x\,dx$

So $I = \int -\frac{1}{2}\sqrt{u}\,du = -\frac{1}{2}\int u^{\frac{1}{2}}\,du$

$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + c$

$= -\frac{1}{3}(1-x^2)^{3/2} + c$

(8) Let $I = \int \cos 2x \cdot (\sin 2x + 3)^2\,dx$

put $u = \sin 2x + 3$, $\therefore \frac{du}{dx} = 2\cos 2x$

$\Rightarrow \frac{1}{2}du = \cos 2x \cdot dx$

So $I = \int \frac{1}{2}u^2\,du = \frac{1}{2}\frac{u^3}{3} + c$

$= \frac{1}{6}(\sin 2x + 3)^3 + c$

(9) Let $I = \int x^2 \cdot (1-x^3)\,dx$

put $u = 1-x^3$, $\therefore \frac{du}{dx} = -3x^2 \Rightarrow -\frac{1}{3}du = x^2\,dx$

So $I = \int -\frac{1}{3}u\,du = -\frac{1}{3}\cdot\frac{1}{2}u^2 + c$

$= -\frac{1}{6}(1-x^3)^2 + c.$

(10) Let $I = \int e^x \sqrt{1+e^x}\, dx$

put $u = 1+e^x$, $\therefore \frac{du}{dx} = e^x \Rightarrow du = e^x\, dx$

So $I = \int \sqrt{u}\, du = \int u^{1/2}\, du = \frac{2}{3} u^{3/2} + c$

$= \frac{2}{3} (1+e^x)^{3/2} + c$

(11) Let $I = \int \cos x \cdot \sin^4 x\, dx$

put $u = \sin x$, $\therefore \frac{du}{dx} = \cos x \Rightarrow du = \cos x\, dx$

$\therefore I = \int u^4\, du = \frac{u^5}{5} + c$

$= \frac{1}{5} \sin^5 x + C$

(12) Let $I = \int \sec^2 x \tan^3 x\, dx$

put $u = \tan x$, $\therefore \frac{du}{dx} = \sec^2 x \Rightarrow du = \sec^2 x\, dx$

So $I = \int u^3\, du = \frac{1}{4} u^4 + c$

$= \frac{1}{4} \tan^4 x + C$

(13) Let $I = \int x^n (1+x^{n+1})^2 \, dx$

put $u = 1 + x^{n+1}$, $\therefore \frac{du}{dx} = (n+1)x^n$

$$\Rightarrow \frac{1}{n+1} du = x^n dx$$

$$\therefore I = \int \frac{1}{n+1} \cdot u^2 \, du = \frac{1}{n+1} \cdot \frac{1}{3} u^3 + c$$

$$= \frac{1}{3(n+1)} (1+x^{n+1})^3 + c$$

(14) Let $I = \int \operatorname{cosec}^2 x \cdot \cot^2 x \, dx$

Put $u = \cot x$, $\therefore \frac{du}{dx} = -\operatorname{cosec}^2 x$

So $-du = \operatorname{cosec}^2 x \, dx$

$$\therefore I = \int -u^2 \, du = -\frac{1}{3} u^3 + c$$

$$= -\frac{1}{3} \cot^3 x + c$$

(15) Let $I = \int \sqrt{x} \cdot \sqrt{1+x^{3/2}} \, dx$

put $u = 1 + x^{3/2}$, so $\frac{du}{dx} = \frac{3}{2} x^{1/2}$

$$\Rightarrow \frac{2}{3} du = \sqrt{x} \cdot dx$$

$$\therefore \quad I = \int \frac{2}{3} \sqrt{u} \, du = \frac{2}{3} \int u^{1/2} \, du$$

$$= \frac{2}{3} \cdot \frac{2}{3} u^{3/2} + c$$

$$= \frac{4}{9} \left(1 + x^{3/2}\right)^{3/2} + c$$

(16) Let $I = \int x^3 (x^4+4)^2 \, dx$

let $u = x^4 + 4$, $\therefore \dfrac{du}{dx} = 4x^3 \Rightarrow \dfrac{1}{4} du = x^3 dx$

So $I = \int \frac{1}{4} u^2 \, du = \frac{1}{4} \cdot \frac{1}{3} u^3 + c$

$$= \frac{1}{12} (x^4+4)^3 + c$$

(17) Let $I = \int e^x (1 - e^x)^3 \, dx$

let $u = 1 - e^x$, $\therefore \dfrac{du}{dx} = -e^x \Rightarrow -du = e^x dx$

So $I = \int -u^3 \, du = -\frac{1}{4} u^4 + c$

$$= -\frac{1}{4} (1 - e^x)^3 + c$$

(18) Let $I = \int \sin\theta \cdot \sqrt{1-\cos\theta} \cdot d\theta$

Let $u = 1 - \cos\theta$, $\therefore \frac{du}{d\theta} = +\sin\theta$

$\Rightarrow du = \sin\theta \, d\theta$

So $I = \int u^{1/2} \, du = \frac{2}{3} u^{3/2} + c$

$= \frac{2}{3} (1 - \cos\theta)^{3/2} + c$

(19) Let $I = \int (x+1)\sqrt{x^2+2x+3} \, dx$

let $u = x^2 + 2x + 3$, so that $\frac{du}{dx} = 2x + 2$

$\Rightarrow \frac{1}{2} du = (x+1) dx$

$\therefore I = \int \frac{1}{2} u^{1/2} \, du = \frac{1}{2} \cdot \frac{2}{3} \cdot u^{3/2} + c$

$= \frac{1}{3} (x^2 + 2x + 3)^{3/2} + c$

(20) Let $I = \int x \cdot e^{x^2+1} \, dx$

let $u = x^2 + 1$, $\therefore \frac{du}{dx} = 2x \Rightarrow \frac{1}{2} du = x \, dx$

$\therefore I = \int \frac{1}{2} e^u \, du = \frac{1}{2} e^u + c = \frac{1}{2} e^{x^2+1} + c$